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|  | **Department of Computer Science**  GSC-221**—Discrete Mathematics**  **Semester 1 Section A,B (Fall2023)** |

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| Assignment N0. 02  Submission Due Date: 3 Nov, 23  Marks 5 |

Instructions: Discuss amongst yourselves, and if you still can’t do it, you can ask me via email or in person. Late submissions are not accepted.

**Case study**

You are walking in a labyrinth and all of a sudden you ﬁnd yourself in front of three possible roads: the road on your left is paved with gold, the one in front of you is paved with marble, while the one on your right is made of small stones. Each street is protected by a guardian. You talk to the guardians and this is what they tell you:

The guardian of the gold street: “This road will bring you straight to the center. Moreover, if the stones take you to the center, then also the marble takes you to the center.” • The guardian of the marble street: “Neither the gold nor the stones will take you to the center.” • The guardian of the stone street: “Follow the gold and you’ll reach the center, follow the marble and you will be lost.”

Given that you know that all the guardians are liars, can you choose a road being sure that it will lead you to the center of the labyrinth? If this is the case, which road you choose?

Provide a propositional language and a set of axioms that formalize the problem and show whether you can choose a road being sure it will lead to the center. Model your solution with the help of rule of inferences.

Identify the given argument is True or False

It follows that if ∃y∀xP(x, y) is true, then ∀x∃yP(x, y) must also be true. However, if ∀x∃yP(x, y) is true, it is not necessary for ∃y∀xP(x, y) to be true.

Support your answer with the help of an example.

Answer: This is a classic example of the difference between the existential and universal quantifiers. Let’s consider a scenario where P(x, y) is defined as “x is a student of y”.

If ∃y∀xP(x, y) is true, it means that there exists a student who studies under every teacher. For example, if we have three teachers A, B, and C, and students 1, 2, 3, and 4, such that:

* Student 1 studies under teachers A, B, and C.
* Student 2 studies under teachers A and B.
* Student 3 studies under teachers A and C.
* Student 4 studies under teacher A.

Then ∃y∀xP(x, y) is true because teacher A has all four students studying under them.

Now let’s consider ∀x∃yP(x, y). This statement means that every student has at least one teacher. In our example above, this statement is true because every student has at least one teacher they study under.

However, it is not necessary for ∃y∀xP(x, y) to be true if ∀x∃yP(x, y) is true. For example, if we have two teachers A and B and two students 1 and 2 such that:

* Student 1 studies under teacher A.
* Student 2 studies under teacher B.

Then ∀x∃yP(x, y) is true because every student has at least one teacher they study under. However, ∃y∀xP(x, y) is false because no teacher has both students studying under them.

Therefore we can conclude that if ∃y∀xP(x, y) is true then ∀x∃yP(x, y) must also be true but the converse is not necessarily true.

**Activity**

**Boolean Searches**

Logical connectives are used extensively in searches of large collections of information, such as indexes of Web pages. Because these searches employ techniques from propositional logic, Links they are called Boolean searches. In Boolean searches, the connective AND is used to match records that contain both of two search terms, the connective OR is used to match one or both of two search terms, and the connective NOT (sometimes written as ANDNOT ) is used to exclude a particular search term. Careful planning of how logical connectives are used is often required when Boolean searches are used to locate information of potential interest. Example 6 illustrates how Boolean searches are carried out.

**Web Page Searching**

Most Web search engines support Boolean searching techniques, which is useful for ﬁnding Web pages about particular subjects. For instance, using Boolean searching to ﬁnd Web pages about universities in New Mexico, we can look for pages matching NEW AND MEXICO AND UNIVERSITIES. The results of this search will include those pages that contain the three words NEW, MEXICO, and UNIVERSITIES. This will include all of the pages of interest, together with others such as a page about new universities in Mexico. (Note that Google, and many other search engines, do require the use of “AND” because such searchExtra Examples engines use all search terms by default.) Most search engines support the use of quotation marks to search for speciﬁc phrases. So, it may be more eﬀective to search for pages matching “NEW MEXICO” AND UNIVERSITIES. Next, to ﬁnd pages that deal with universities in New Mexico or Arizona, we can search for pages matching (NEWANDMEXICOORARIZONA)ANDUNIVERSITIES. (Note: Here the AND operator takes precedence over the OR operator. Also, in Google, the terms used for this search would be NEW MEXICO OR ARIZONA.) The results of this search will include all pages that contain the word UNIVERSITIES and either both the words NEW and MEXICO or the word ARIZONA. Again, pages besides those of interest will be listed. Finally, to ﬁnd Web pages that deal with universities in Mexico (and not New Mexico), we might ﬁrst look for pages matching MEXICO AND UNIVERSITIES, but because the results of this search will include pages about universities in New Mexico, as well as universities in Mexico, it might be better to search for pages matching (MEXICO AND UNIVERSITIES) NOT NEW. The results of this search include pages that contain both the words MEXICO and UNIVERSITIES but do not contain the word NEW. (In Google, and many other search engines, the word “NOT” is replaced by the symbol “-”. In Google, the terms used for this last search would be MEXICO UNIVERSITIES -NEW.)

With the reference of above information, search the material on google and show the screenshots of your results, such as

